

# Effective Sextic Superpotential and $B - L$ violation in NMSGUT

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**Abstract.** We list operators of the superpotential of the effective MSSM that emerges from the NMSGUT up to sextic degree. We give illustrative expressions for the coefficients in terms of NMSGUT parameters. We also estimate the impact of GUT scale threshold corrections on these effective operators in view of the demonstration that  $B$  violation via quartic superpotential terms can be suppressed to acceptable levels after including such corrections in the NMSGUT. We find a novel  $B, B - L$  violating quintic operator that leads to the decay mode  $n \rightarrow e^- K^+$ . We also remark that the threshold corrections to the Type I seesaw mechanism make the deviation of right handed neutrino masses from the GUT scale more natural while Type II seesaw neutrino masses, which earlier tended to utterly negligible receive threshold enhancement. Our results are of relevance for analyzing  $B - L$  violating operator based, sphaleron safe, Baryogenesis.

**Keywords.** NMSGUT, SO(10), Effective MSSM, Threshold corrections, coupling enhancement.

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## 1. Introduction

Despite various open questions of principle, such as a mechanism to ensure light Standard Model (SM) Higgs doublets, and still without any direct experimental proof, particularly proton decay, Grand Unification models based on SO(10) remain the most transparent framework to think about beyond standard model (BSM) physics. In such grand unification theories (GUTs) neutrino masses are tightly connected to rest of the fermion masses of the SM and other high scale parameters. They explain charge quantization, give gauge and matter (quark-lepton) unification and predict important exotic processes (proton decay) and parameters (neutrino masses and mixing).

The effective field theory (EFT) of non-renormalizable higher dimensional operators (HDO) formed from the fields of the SM plays an essential role in BSM studies. While the bottom-up approach to the effective theory makes fewer speculations it also gives no clue regarding the scale of new physics and the sizes of the coupling coefficients of HDOs; or rather it fails to utilize the few but strong hints about BSM scales available from the size of neutrino masses and the unification of couplings. However given a viable GUT, specially a supersymmetric (SUSY) one, it is straightforward to derive the HDOs that correct the leading order renormalizable (minimal supersymmetric) standard model, (MS)SM, that arises from the GUT when superheavy

fields are set to zero, simply by solving the (algebraic) equations for the heavy (super)fields in the approximation that their momenta are negligible compared to their masses. Exotic operators that violate accurate but apparently accidental symmetries of Standard Model, such as Baryon ( $B$ ) and Lepton ( $L$ ) numbers are obviously of most interest for developing expectations regarding the experimental implication of any particular UV completion of the SM. HDOs which break  $B$  and  $L$  symmetry while preserving  $B - L$  are familiar consequences of GUTs. On the other hand Majorana neutrino masses ( $\Delta L \neq 0$ ) require breaking of  $B - L$  symmetry and in fact arise in renormalizable models of Type I and Type II seesaw via vacuum expectation values (VEVs) of  $B - L$  non-singlet fields.  $B - L$  violating processes may be particularly important for GUT scale baryogenesis because a baryon asymmetry produced via  $B - L$  conserving operators is liable to washout via Electroweak sphaleron processes that are unsuppressed above  $M_W$ . On the other hand baryon asymmetry generated via  $B - L$  violating operators that also violate  $B$  can generate a Baryon asymmetry that survives Spahleron washout [1]. This revives the possibility that the observed baryon asymmetry may arise at GUT scales rather than by sphaleron reprocessing of the Lepton asymmetry generated via  $B - L$  violating decay of heavy righthanded (RH) neutrinos [2]. It is interesting that this possibility is proposed in the context of the same theories with gauged  $B - L$  (such as SO(10) GUTs) that

provide a natural context for Type I and Type II Seesaw masses.

Minimal Supersymmetric SO(10) Grand Unification Theory (MSGUT) [3–5] has been developed over a long period into a completely realistic scenario compatible with known processes, data and structures of physics up to several hundred GeV, including the critical BSM phenomenon of neutrino masses, to an extent that is not rivaled by any other model. Moreover the solubility of the spontaneous symmetry breaking (SSB) at GUT scales permits expression of SM data in terms of GUT couplings, explicit evaluation of GUT scale threshold corrections and determination of viable  $B$  violation operators. There are a number of other attractions of the New-MSGUT (NMSGUT): Minimal number of parameters, R-parity preserving effective MSSM giving viable SUSY WIMP Dark Matter, large soft trilinear ( $A_0$ ) indications in advance of Higgs discovery [6] etc. Thus the evaluation of the exotic operators for this theory is of relevance. An attempt in this direction for a closely related model which modifies the NMSGUT just by adding an additional **10**-plet (and thus shares the same SSB structure at high scales) has already appeared [7]. It uses the rather unwieldy decomposition of SO(10) invariants via  $SU(5) \times U(1)$  maximal subgroup of SO(10). However the NMSGUT analysis mentioned above was via the more symmetric, and thus somewhat more convenient and transparent, decomposition of SO(10) in terms of the Pati-Salam (PS) group  $G_{PS} \equiv SU(2)_L \times SU(2)_R \times SU(4)$ . Thus an evaluation of the effective operators and their coefficients in terms of the GUT coupling using this method is both possible (given the detailed decompositions we generated during our previous calculations [8, 9, 6, 10]) and required considering the need to evaluate the workability of the NMSGUT as a comprehensive unification of particle physics i.e. beyond just fitting of SM data and compatibility with exclusion limits on exotic processes. In this proceeding we report on our calculation of the the effective operators possible in the NMSGUT superpotential up to terms sextic in the chiral fields and on their coefficients in terms of GUT parameters up to terms quintic. We also found one new operator which was missing in the literature which is relevant to the study of nucleon decay process in  $B-L$  violating baryogenesis.

Techniques for computing the decomposition of SO(10) invariants in terms of PS along with coefficients of dimension 5 operators for  $B$ ,  $L$  violation were presented in [8, 6]. The complete spectrum was presented in [9] for MSGUT and in [6] for NMSGUT. See [11, 12] for related spectrum calculations. Decomposition of the NMSGUT invariants which contain Higgs doublets as one of the fields in the invariants were evaluated for the

computations in [10] and are thus available to us.

In Section 2. we discuss the effective MSSM theory emerging from NMSGUT. In Section 3. we will recapitulate the impact of GUT scale threshold corrections on the strength of coefficients of operators and their impact on proton lifetime estimation. In Section 4. we discuss impact of threshold correction on  $B-L$  violating processes. In Section 5. we conclude. Tables of effective superpotential operators are provided in the Appendix.

## 2. Effective MSSM from NMSGUT

Effective theories derived by integrating out heavy fields are familiar from the basic example of the Fermi theory of beta decay that arises from the Electro-weak theory by integrating out the heavy  $W, Z$  gauge bosons. The most familiar paradigm for the effective Lagrangian due to integrating out heavy matter fields is Weinberg's [13] unique,  $L$  and  $B-L$  violating, dimension five, Majorana mass term for left handed neutrinos. It arises from integrating out right handed neutrinos with large ( $B-L$  violating) Majorana masses :  $m_{\nu_L} \sim Y_{AB}^v L_A H L_B H / M_{\nu_R}$ . If the neutrino Yukawa couplings  $Y_{AB}^v$  are similar in size to the Yukawa couplings for charged fermions of their generation, as expected from GUTs, specially SO(10) GUTs, then the required scale of new physics indicated by the measured neutrino masses is near the GUT scale. The scale of new physics can be lowered if these couplings are much smaller. If the leading order contributions are completely absent, or when violation of superselection rules applicable to the lowest dimension effective operators is relevant (as for  $B$  violation or when  $B-L$  violating operators enable sphaleron-safe baryogenesis) higher dimensional operators may be significant and must be considered.

Neutrino masses and mixing explained through seesaw mechanism assume a Majorana nature for neutrinos. On the other hand it is still perfectly feasible that right handed neutrinos are in fact light particles and that neutrino masses are of Dirac type. Oscillation experiments cannot distinguish between these possibilities. Other  $L$  and  $B-L$  violating processes such as neutrinoless beta decay ( $0\nu\beta\beta$ ) - can however discriminate between Majorana and Dirac neutrinos. The corresponding effective operator for  $0\nu\beta\beta$  is at least dim-9. Therefore the scale of new physics can be around the collider search scale without any serious fine tuning in the parameters in some models and hence can be probed directly [14].  $B$  and  $B-L$  violating process like  $n - \bar{n}$  oscillation [15–17] emerge from another dim-9 effective operator. However for high scale  $B-L$  violation, as is the rule for SUSY GUTs, such operators are highly

suppressed.

The NMSGUT is a SUSY GUT model based on SO(10) gauge symmetry with all the fermions (including right handed neutrino) of one generation residing in a single 16-plet spinor supermultiplet of Spin(10). Higgs fields which break gauge symmetry and give masses reside in chiral supermultiplets that are **10**, **120**, **126** (+**126**), **210** representations of SO(10). Due to the strong constraints on possible renormalizable SO(10) cubic invariants this model enjoys a minimal number of parameters versus all competing viable models.

The complete NMSGUT superpotential is [3, 8, 9, 6],

$$\begin{aligned} \mathcal{W} = & \left( h_{AB} \mathbf{10} + \frac{f_{AB}}{5!} \overline{\mathbf{126}} + \frac{g_{AB}}{3!} \mathbf{120} \right) \mathbf{16}_A \mathbf{16}_B \\ & + \frac{M_H}{2} \mathbf{10}^2 + \frac{m_\Theta}{2(3!)} \mathbf{120}^2 + \frac{M_\Sigma}{5!} \mathbf{126} \cdot \overline{\mathbf{126}} \\ & + \frac{M_\Phi}{4!} \mathbf{210} \cdot \mathbf{210} + \frac{\lambda}{4!} \mathbf{210}^3 + \frac{\eta}{4!} \mathbf{210} \cdot \mathbf{126} \cdot \overline{\mathbf{126}} \\ & + \frac{1}{2(3!)} \mathbf{120} \cdot \mathbf{210} \cdot (\zeta \mathbf{126} + \bar{\zeta} \overline{\mathbf{126}}) + \frac{\kappa}{3!} \mathbf{10} \cdot \mathbf{120} \cdot \mathbf{210} \\ & + \frac{1}{4!} \mathbf{10} \cdot \mathbf{210} \cdot (\gamma \mathbf{126} + \bar{\gamma} \overline{\mathbf{126}}) + \frac{\rho}{4!} \mathbf{120} \cdot \mathbf{120} \cdot \mathbf{210} \quad (1) \end{aligned}$$

where  $A, B = 1, 2, 3$ ,  $h = h^T$ ,  $f = f^T$  and  $g = -g^T$ . The decomposition of these representations into SM irreps gives 26 types of distinct representations which were therefore conveniently labelled alphabetically [8]. The decomposition of tensorial representations is detailed in [8, 9, 6] together with their mass matrices for mixed and unmixed states. We will adopt the same nomenclature here.

For deriving the effective superpotential it is convenient to divide fields into three categories according to their coupling patterns and masses after SSB : **Green** fields are light with masses of order  $M_Z$  or less and populate the effective theory. **Red** fields have large masses  $\gg M_Z$  but couple with fields in the matter 16-plets. **Blue** fields do not couple with 16-plets and are also superheavy. The pair of MSSM Higgs doublets which is light is thus **Green** and rest (five) of the Higgs type doublet pairs are **Red**. Similarly the heavy right handed neutrino is **Red**. The Higgs (**10**, **120**, **126**) coupled to matter fields are called fermion mass (FM) Higgs and the rest (**210**) are Adjoint mode (AM) Higgs. The SM decomposition of  $\mathcal{W}_{FM}$  is given in eq.(58,60) of [9] and eq.(5.5) of [6]. The superpotential has **Green** - **Green** - **Green** (GGG) terms containing SM fermions with either of the light Higgs pair, **Green** - **Green** - **Red** (GGR) containing two SM fermions and one heavy FM higgs field or SM fermion-light Higgs-RH neutrino.  $\mathcal{W}_{AM}$  contributes a large number of cubic **Blue** fields interacting with **Blue** or **Red** fields as well as BB and RR masses, but also contains cubic interactions with **Green**

Green	$\bar{u}, \bar{d}, Q, \bar{e}, L, H, \bar{H}$
Red	$\bar{\nu}, A, C, D, E, F, G, h, J, K, L, M, N, O, P, t, W$
Blue	$B, I, Q, R, S, U, V, X, Y, Z$

**Table 1.** Nature of the fields

(light Higgs fields). However the light Higgs (by definition) lack superpotential mass terms.

$$\begin{aligned} \mathcal{W}_{FM}^{GGG} = & \bar{H} \left[ -\frac{4}{\sqrt{2}} h_{AB} U_{11}^{h\dagger} (\bar{d}_A Q_B + \bar{e}_A L_B) \right. \\ & + 4\sqrt{2} \frac{i}{\sqrt{3}} f_{AB} U_{12}^{h\dagger} (\bar{d}_A Q_B - 3\bar{e}_A L_B) \\ & + 2\sqrt{2} g_{AB} U_{15}^{h\dagger} (\bar{d}_A Q_B + \bar{e}_A L_B) \\ & \left. - 2\sqrt{2} g_{AB} \frac{i}{\sqrt{3}} U_{16}^{h\dagger} (\bar{d}_A Q_B - 3\bar{e}_A L_B) \right] \\ & + H(\bar{u}_A Q_B) \left[ 2\sqrt{2} h_{AB} V_{11}^h - 4\sqrt{2} \frac{i}{\sqrt{3}} f_{AB} V_{21}^h \right. \\ & \left. - 2\sqrt{2} g_{AB} V_{51}^h + 2\sqrt{2} g_{AB} \frac{i}{\sqrt{3}} V_{61}^h \right] \quad (2) \end{aligned}$$

The matrices  $U^\Phi, V^\Phi$  diagonalize Higgs field superpotential masses as  $U^\dagger M V = M_{diag}$  with the alphabetic superscripts indicating which of the 13 mixing types is involved (see [6, 10] for details).  $\mathcal{W}_{FM}^{GGR}$  has two light matter fields and one heavy Higgs (**Red**) field or else light matter, light Higgs, and RH neutrino:

$$\begin{aligned} \mathcal{W}_{FM}^{GGR} = & \Gamma_{jAB}^{\bar{t}u\bar{d}} \bar{t}_j \epsilon \bar{u}_A \bar{d}_B + \Gamma_{jAB}^{\bar{t}QL} \bar{t}_j Q_A L_B + \Gamma_{jAB}^{\bar{t}QQ} \bar{t}_j \frac{\epsilon}{2} Q_A Q_B \\ & + \Gamma_{jAB}^{\bar{t}u\bar{e}} \bar{t}_j \bar{u}_A \bar{e}_B + \Gamma_{jAB}^{\bar{C}dQ} \bar{C}_j \bar{d}_A Q_B + \Gamma_{jAB}^{\bar{C}uQ} \bar{C}_j \bar{u}_A Q_B \\ & + \Gamma_{jAB}^{\bar{D}uL} \bar{D}_j \bar{u}_A L_B + \Gamma_{jAB}^{\bar{D}eQ} \bar{D}_j \bar{e}_A Q_B + \Gamma_{jAB}^{\bar{E}dL} \bar{E}_j \bar{d}_A L_B \\ & + \Gamma_{AB}^{\bar{A}e\bar{e}} \bar{A} \bar{e}_A \bar{e}_B + \Gamma_{AB}^{\bar{W}QQ} \bar{W} Q_A Q_B + \Gamma_{jAB}^{\bar{P}QL} \bar{P}_j Q_A L_B \\ & + \Gamma_{jAB}^{\bar{P}QQ} \bar{P}_j Q_A Q_B + \Gamma_{AB}^{\bar{O}LL} \bar{O} L_A L_B + \Gamma_{jAB}^{\bar{K}d\bar{e}} \bar{K}_j \bar{d}_A \bar{e}_B \\ & + \Gamma_{jAB}^{\bar{K}u\bar{u}} \bar{K}_j \bar{u}_A \bar{u}_B + \Gamma_{jAB}^{\bar{L}u\bar{d}} \bar{L}_j \bar{u}_A \bar{d}_B + \Gamma_{jAB}^{\bar{L}QQ} \bar{L}_j Q_A Q_B \\ & + \Gamma_{jAB}^{\bar{F}LL} \bar{F}_j L_A L_B + \Gamma_{jAB}^{\bar{J}d\bar{d}} \bar{J}_j \bar{d}_A \bar{d}_B + \Gamma_{\bar{k}AB}^{\bar{h}dQ} \bar{h}_{\bar{k}} \bar{d}_A Q_B \\ & + \Gamma_{\bar{k}AB}^{\bar{h}eL} \bar{h}_{\bar{k}} \bar{e}_A L_B + \Gamma_{\bar{k}AB}^{\bar{h}uQ} \bar{h}_{\bar{k}} \bar{u}_A Q_B + \Gamma_{AB}^{\bar{h}\nu L} \bar{H} \bar{\nu}_A L_B \\ & + \Gamma_{AB}^{\bar{N}d\bar{d}} \bar{N} \bar{d}_A \bar{d}_B + \Gamma_{AB}^{\bar{M}u\bar{u}} \bar{M} \bar{u}_A \bar{u}_B \quad (3) \end{aligned}$$

The coefficients  $\Gamma_{abc}^{ABC}$  are expressed in terms of GUT

parameters by equations like

$$\begin{aligned}\Gamma_{jAB}^{\bar{t}u\bar{d}} &= \left[ 2\sqrt{2}h_{AB}U_{j1}^{t\dagger} - 4\sqrt{2}f_{AB}U_{j2}^{t\dagger} + 4ig_{AB}U_{j7}^{t\dagger} \right] \\ \Gamma_{AB}^{M\bar{u}\bar{u}} &= 4\sqrt{2}f_{AB}\end{aligned}\quad (4)$$

and complete expressions will be reported in [18]. The indices  $A, B$  run over three flavor generations of fermions. The hat over the indices of the **Red** fields is to indicate that they are in mass diagonal basis. The index over heavy Higgs doublets  $\hat{h}$  ( $\bar{\hat{h}}$ ) starts from 2 and is distinguished by a bar over it.

Similarly  $\mathcal{W}_{FM}^{GRR}$  has one **Green** and two **Red** fields out of which one is heavy neutrino.

$$\begin{aligned}\mathcal{W}_{FM}^{GRR} &= \Lambda_{jAB}^{t\bar{d}\bar{v}} \bar{t}_j \bar{v}_B \bar{d}_A + \Lambda_{jAB}^{\bar{E}\bar{v}Q} \bar{E}_j \bar{v}_A Q_B + \Lambda_{jAB}^{\bar{F}\bar{e}\bar{v}} \bar{F}_j \bar{e}_A \bar{v}_B \\ &+ \Lambda_{jAB}^{J\bar{u}\bar{v}} J_j \bar{u}_A \bar{v}_B + \Lambda_{\hat{k}AB}^{h\bar{v}L} h_{\hat{k}} \bar{v}_A L_B\end{aligned}\quad (5)$$

The coefficients  $\Lambda_{abc}^{ABC}$  are expressed [18] in terms of GUT parameters by equations like

$$\begin{aligned}\Lambda_{jAB}^{t\bar{d}\bar{v}} &= \left[ -2\sqrt{2}h_{AB}V_{1j}^t + 4\sqrt{2}f_{AB}V_{2j}^t - 8if_{AB}V_{4j}^t \right. \\ &\quad \left. + 4g_{AB}V_{6j}^t + 4ig_{AB}V_{7j}^t \right]\end{aligned}\quad (6)$$

$\mathcal{W}_{FM}^{RRR}$  has three **Red** fields and therefore requires two RH neutrinos.

$$\mathcal{W}_{FM}^{RRR} = -8if_{AB}G_5 \bar{v}_A \bar{v}_B \quad (7)$$

Similarly in the adjoint mode we have

$$\begin{aligned}\mathcal{W}_{AM}^{GGR} &= \Upsilon^O H\bar{H}O + \Upsilon^{\bar{O}} \bar{H}\bar{H}\bar{O} + \Upsilon^S H\bar{H}S \\ &+ \Upsilon_j^G H\bar{H}G_j + \Upsilon_j^{\bar{F}} H\bar{H}\bar{F}_j + \Upsilon_j^F \bar{H}\bar{H}F_j\end{aligned}\quad (8)$$

Note that, as first shown in [9] the  $H\bar{H}O$ ,  $\bar{H}\bar{H}\bar{O}$  novel terms are the basis of the Type II seesaw in the MSGUT due to the  $RGG$  term  $\bar{O}LL$  in eq.(3). The  $M_O\bar{O}O$  mass term implies  $\langle \bar{O} \rangle \sim \langle HH \rangle / M_O$  leading to Type II masses for neutrinos. The coefficients  $\Upsilon_a^A$  are expressed [18] in terms of GUT parameters by equations like

$$\Upsilon^O = \eta 2\sqrt{3}V_{21}^h V_{41}^h + i\zeta\sqrt{3}V_{61}^h V_{41}^h + \zeta V_{51}^h V_{41}^h \quad (9)$$

Similarly

$$\begin{aligned}\mathcal{W}_{AM}^{GRR} &= \Omega_{\hat{k}j}^{h\bar{F}} H\hat{h}_{\hat{k}} \bar{F}_j + \Omega_{\hat{k}j}^{\bar{h}F} \bar{H}\hat{h}_{\hat{k}} F_j + \Omega_{ij}^{E\bar{J}} H\bar{E}_i \bar{J}_j \\ &+ \Omega_{ij}^{\bar{E}J} \bar{H}\bar{E}_i J_j + \Omega_{ij}^{H\bar{t}} H\bar{t}_i \bar{E}_j + \Omega_{ij}^{\bar{H}t} \bar{H}\bar{t}_i E_j \\ &+ \Omega_{ij}^{\bar{E}P} H\bar{P}_i \bar{E}_j + \Omega_{ij}^{E\bar{P}} \bar{H}\bar{P}_i E_j + \Omega_{\hat{k}j}^{\bar{h}G} H\hat{h}_{\hat{k}} G_j \\ &+ \Omega_{\hat{k}j}^{h\bar{G}} H\hat{h}_{\hat{k}} \bar{G}_j + \Omega_{ij}^{J\bar{D}} HJ_i \bar{D}_j + \Omega_{ij}^{\bar{J}D} \bar{H}\bar{J}_i D_j \\ &+ \Omega_{\hat{k}}^{\bar{h}S} H\hat{h}_{\hat{k}} S + \Omega_{\hat{k}}^{h\bar{S}} H\hat{h}_{\hat{k}} \bar{S} + \Omega_{\hat{k}}^{hO} H\hat{h}_{\hat{k}} O \\ &+ \Omega_{\hat{k}}^{\bar{h}\bar{O}} \bar{H}\hat{h}_{\hat{k}} \bar{O}\end{aligned}\quad (10)$$

The coefficients  $\Omega_{ab}^{AB}$  are expressed [18] in terms of GUT parameters by equations like

$$\begin{aligned}\Omega_{j\hat{k}}^{h\bar{F}} &= \frac{4\eta}{\sqrt{3}}V_{21}^h V_{3j}^h U_{\hat{k}2}^{F\dagger} + i\eta 2\sqrt{3}V_{31}^h V_{4j}^h U_{\hat{k}1}^{F\dagger} \\ &- \frac{i\rho}{3}V_{51}^h V_{6j}^h U_{\hat{k}2}^{F\dagger} + \frac{i\rho}{\sqrt{3}}V_{61}^h V_{4j}^h U_{\hat{k}4}^{F\dagger} \\ &+ \frac{2i\zeta}{\sqrt{3}}V_{61}^h V_{3j}^h U_{\hat{k}2}^{F\dagger} + \zeta V_{51}^h V_{3j}^h U_{\hat{k}2}^{F\dagger} \\ &+ \zeta\sqrt{3}V_{31}^h V_{4j}^h U_{\hat{k}4}^{F\dagger} + \zeta V_{51}^h V_{2j}^h U_{\hat{k}2}^{F\dagger} \\ &+ \frac{2i\bar{\zeta}}{\sqrt{3}}V_{61}^h V_{2j}^h U_{\hat{k}2}^{F\dagger} - \bar{\zeta}\sqrt{\frac{3}{2}}V_{61}^h V_{4j}^h U_{\hat{k}1}^{F\dagger} \\ &+ \bar{\zeta}\sqrt{3}V_{21}^h V_{4j}^h U_{\hat{k}4}^{F\dagger} - i\bar{\zeta}V_{51}^h V_{4j}^h U_{\hat{k}1}^{F\dagger} \\ &+ \left( V_{m1}^h V_{nj}^h \leftrightarrow V_{n1}^h V_{mj}^h \right)\end{aligned}\quad (11)$$

Similarly we will have  $\mathcal{W}_{AM}^{RRR}$ ,  $\mathcal{W}_{AM}^{GRB}$  and  $\mathcal{W}_{AM}^{RRB}$ . In AM contributions  $G$  can only be a light Higgs doublet. Going beyond  $\mathcal{W}_{AM}^{GRB}$  is not required for obtaining the sextic effective potential, and the relevant operators would be even more hopelessly suppressed. Mass terms of heavy fields are not given here because they acquire masses both from mass terms and the super-heavy VEVs of the Higgs fields and the mass matrices for all such heavy fields are given in [9, 6]. Now using the equations of motion we sequentially integrate out **Blue** fields in terms of **Red-**Green**** fields, **Red** fields in terms of **Green-**Green**** fields. At the end of this section we have shown that the superpotential terms with **Blue** fields give raise to sextic or higher dimensional effective operators. The quartic and quintic effective operator purely emerge from the superpotential terms with one or two **Red** fields. Plugging them back in to the superpotential terms leads to effective **Green** operators. For example the low energy equations of motion for **Red** fields like RH neutrino (see eq.(2)) and  $F$  (see eq.(5) and (8)) give :

$$\bar{v}_A = -M_{AB}^{\bar{v}}{}^{-1} \left[ \Gamma_{BC}^{h\bar{v}L} H L_C \right] \quad (12)$$

$$F_j = -\mathcal{F}_{jj} \left[ \Upsilon_j^{\bar{F}} H H + \Lambda_{jAB}^{\bar{F}\bar{e}\bar{v}} \bar{e}_A \bar{v}_B \right] \quad (13)$$

where  $\mathcal{F}$  is the inverse of the diagonalized  $4 \times 4$  mass matrix for  $\bar{F} - F$  type fields, and other coefficients are like

$$\begin{aligned}\Gamma_{AB}^{h\bar{v}L} &= 2\sqrt{2}h_{AB}V_{11}^h + 4i\sqrt{6}f_{AB}V_{21}^h \\ &- 2\sqrt{2}g_{AB}V_{51}^h - 2i\sqrt{6}g_{AB}V_{61}^h.\end{aligned}\quad (14)$$

Although we have called both  $\bar{v}$  and  $A, C \dots$  **Red** (see Table 1) the  $\bar{v}$  masses lie in  $10^7 - 10^{13}$  GeV and are thus

much lighter than typical GUT scales. This difference in scales is set by the requirement that the Type I seesaw masses match the observed neutrino masses. Since in the MSGUT we break  $SO(10)$  to the SM gauge group in a single step, and integrating out right handed neutrinos in stages increases the complexity without drastic effects on the neutrino masses we integrate out all the right handed neutrinos at the GUT breaking scale as well. An improved treatment [19] would first integrate out only GUT scale mass particles calculate renormalization group flow corrected couplings at the heaviest neutrino scale, integrate out the heaviest neutrino and repeat till the MSSM corrected by the  $d = 5$  Weinberg operator for Majorana neutrino masses was reached. While this procedure may be followed in numerical work, our intention here is to indicate the effects of GUT scale thresholds. We shall show below that the same GUT threshold corrections which suppress proton decay facilitate the separation of right handed neutrino masses from GUT scales and hence more natural. Terms coming from  $\bar{\nu}$  substitution are more important than generic **Red** Higgs fields. Substituting  $\bar{\nu}$  from eq.(12) in eq.(13) we see that term with coefficient  $\Lambda_{jAB}^{\bar{F}\bar{\nu}}$  is subleading compared to the term with coefficient  $\Upsilon_j^{\bar{F}}$ , and will contribute to quintic operator. Similarly, we can see from eq.(5), the fields  $\bar{E}, \bar{I}, \bar{J}$  and  $\bar{h}$  will contain such  $\bar{\nu}$  dependent terms.

Now, plugging back these results in  $\mathcal{W}_{FM}^{GGR}$  (eq.(3)) and  $\mathcal{W}_{AM}^{GGR}$  (eq.(8)) we get the following quartic superpotential at the leading order

$$\begin{aligned}\mathcal{W}^4 &= \mathcal{W}_{(1,1)}^4 + \mathcal{W}_{(0,0)}^4 + \mathcal{W}_{(-1,-1)}^4 + \mathcal{W}_{(0,2)}^4 \\ \mathcal{W}_{(1,1)}^4 &= L_{ABCD} \frac{\epsilon}{2} Q_A Q_B Q_C L_D \\ \mathcal{W}_{(-1,-1)}^4 &= R_{ABCD} \epsilon \bar{e}_A \bar{u}_B \bar{u}_C \bar{d}_D \\ \mathcal{W}_{(0,2)}^4 &= P_{AB}^I (L_A L_B) (HH) + P_{AB}^{II} (L_A H) (L_B H) \\ \mathcal{W}_{(0,0)}^4 &= Q_{ABCD}^I (Q_A L_B) \bar{u}_C \bar{e}_D + Q_{ABCD}^{II} (Q_A \bar{d}_D) (Q_B \bar{u}_C) \\ &\quad + Q_{ABCD}^{III} (\bar{H} H) (\bar{H} H) + Q_{ABCD}^{IV} (\bar{H} H) (HH) \quad (15)\end{aligned}$$

where subscripts on the superpotential components denote the  $(B, L)$  numbers. The coefficients  $L_{ABCD}$  and  $R_{ABCD}$  are given in eq.(5.7 & 5.8) of [6] and other coefficients are like

$$P_{AB}^I = -\Gamma_{jAB}^{FLL} \mathcal{F}_{jj} \Upsilon_j^F, \quad (16)$$

where  $\Gamma_{jAB}^{FLL} = -(2\sqrt{2}g_{AB})V_{4j}^F$  and

$$\begin{aligned}\Upsilon_j^F &= \left[ -\frac{4\eta}{\sqrt{3}} U_{12}^{h\dagger} U_{13}^{h\dagger} V_{2j}^F - i\eta 2\sqrt{3} U_{12}^{h\dagger} U_{14}^{h\dagger} V_{1j}^F \right. \\ &\quad + \frac{i\rho}{3} U_{15}^{h\dagger} U_{16}^{h\dagger} V_{2j}^F - \frac{i\rho}{\sqrt{3}} U_{16}^{h\dagger} U_{14}^{h\dagger} V_{4j}^F - \frac{2i\zeta}{\sqrt{3}} U_{16}^{h\dagger} U_{13}^{h\dagger} V_{2j}^F \\ &\quad + \zeta \sqrt{3} U_{13}^{h\dagger} U_{14}^{h\dagger} V_{4j}^F + i\zeta U_{15}^{h\dagger} U_{14}^{h\dagger} V_{1j}^F - \zeta U_{15}^{h\dagger} U_{13}^{h\dagger} V_{2j}^F \\ &\quad - \zeta \sqrt{3} U_{16}^{h\dagger} U_{14}^{h\dagger} V_{1j}^F - \frac{2i\bar{\zeta}}{\sqrt{3}} U_{16}^{h\dagger} U_{12}^{h\dagger} V_{2j}^F - \bar{\zeta} U_{12}^{h\dagger} U_{15}^{h\dagger} V_{2j}^F \\ &\quad \left. + \bar{\zeta} \sqrt{3} U_{12}^{h\dagger} U_{14}^{h\dagger} V_{4j}^F + \bar{\zeta} \sqrt{3} U_{12}^{h\dagger} U_{14}^{h\dagger} V_{4j}^F \right] \quad (17)\end{aligned}$$

and similarly we can find  $P_{AB}^{II}$  and  $Q_{ABCD}^{I,II,III,IV}$  [18]. As explained above, in the dim-4 superpotential  $\mathcal{W}_{(0,2)}^{(4)}$  there exist operators which are Type I and II seesaw terms and violate  $B - L$ . The operator in  $\mathcal{W}_{FM}^{GGR} + \mathcal{W}_{AM}^{GGR}$  with fields  $\bar{F}, \bar{E}, \bar{I}, \bar{J}, \bar{h}$  will also give rise to contributions to quintic operators since  $\bar{F}, \bar{E}, \bar{I}, \bar{J}, \bar{h}$  are  $O(G^3)$  through  $\bar{\nu}$ .

The other quintic effective superpotential terms arise from integrating out two **Red** fields from  $\mathcal{W}_{FM}^{GRR} + \mathcal{W}_{AM}^{GRR}$ . There are three types of quintic operators:

$$\begin{aligned}\mathcal{W}^5 &= \mathcal{W}_{(-1,1)}^5 + \mathcal{W}_{(0,2)}^5 + \mathcal{W}_{(0,0)}^5 \\ \mathcal{W}_{(-1,1)}^5 &= F_{ABCD}^I (HL_A) \epsilon \bar{d}_B \bar{d}_C \bar{u}_D + F_{ABCD}^{II} (\bar{H} L_A) \epsilon \bar{d}_B \bar{d}_C \bar{d}_D \\ \mathcal{W}_{(0,2)}^5 &= G_{ABCD}^I \bar{d}_A (HQ_B) (L_C L_D) + G_{ABCD}^{II} \bar{d}_A (HL_B) (Q_C L_D) \\ &\quad + G_{ABCD}^{III} \bar{e}_A (HL_B) (L_C L_D) \\ \mathcal{W}_{(0,0)}^5 &= H_{AB}^I \bar{e}_A (HL_B) (\bar{H} \bar{H}) + H_{AB}^{II} \bar{e}_A (\bar{H} L_B) (H \bar{H}) \\ &\quad + H_{AB}^{III} \bar{u}_A (\bar{H} Q_B) (HH) + H_{AB}^{IV} \bar{u}_A (HQ_B) (H \bar{H}) \\ &\quad + H_{AB}^V \bar{d}_A (\bar{H} Q_B) (H \bar{H}) + H_{AB}^{VI} \bar{d}_A (HQ_B) (\bar{H} \bar{H}) \quad (18)\end{aligned}$$

and the coefficients are like

$$\begin{aligned}F_{ABCD}^I &= \left[ \Gamma_{jDC}^{\bar{u}\bar{d}} \mathcal{T}_{jj} \Lambda_{jBE}^{d\bar{\nu}} + \Gamma_{jBC}^{\bar{d}\bar{d}} \mathcal{J}_{jj} \Lambda_{jDE}^{j\bar{\nu}} \right] (M_{\bar{\nu}}^{-1})_{EF} \Gamma_{FA}^{h\bar{\nu}L} \\ &\quad - \left[ \Omega_{ij}^{J\bar{D}} \mathcal{J}_{ii} \Gamma_{iBC}^{\bar{J}\bar{d}} \mathcal{D}_{jj} \Gamma_{jDA}^{D\bar{u}L} \right] - \left[ \Omega_{ij}^{\bar{E}} \mathcal{T}_{ii} \Gamma_{iDB}^{\bar{E}\bar{u}} \mathcal{E}_{jj} \Gamma_{jCA}^{E\bar{d}L} \right] \\ F_{ABCD}^{II} &= \left[ -\Omega_{ij}^{\bar{E}J} \mathcal{E}_{ii} \Gamma_{iBA}^{E\bar{d}L} \mathcal{J}_{jj} \Gamma_{jCD}^{\bar{J}\bar{d}} \right] \quad (19)\end{aligned}$$

where  $\mathcal{T}, \mathcal{J}, \mathcal{D}$  and  $\mathcal{E}$  are the inverse of mass matrices  $M_I, M_J, M_D$  and  $M_E$  respectively. Since  $M_{\bar{\nu}} \ll M_{I,J,D,E}$ , the first term of coefficient  $F_{ABCD}^I$  in eq.(19) is most significant. Patterns of the coefficients  $\Gamma, \Lambda$  and  $\Omega$  are given in eq.(4, 14), (6) and (11).

Most interestingly, here we have  $B, L$  and  $B - L$  violating operators  $\mathcal{W}_{(-1,1)}^5$ . They open  $B - L$  violating channels to nucleon decay. The operator  $(HL_A) \epsilon \bar{d}_B \bar{d}_C \bar{u}_D$  has a contribution from  $\mathcal{W}_{FM}^{GRR}$  with one **Red** field being the right handed neutrino, and was discussed in [1, 7]. It also gets a contribution from  $\mathcal{W}_{AM}^{GRR}$  through  $H\bar{J}\bar{D}$  and  $H\bar{I}\bar{E}$  terms (see eq.(8)). In addition to that we have a new quintic operator  $(\bar{H} L_A) \epsilon \bar{d}_B \bar{d}_C \bar{d}_D$  - with coefficient



$F_{ABCD}^{II}$  as given in eq.(19) - which arises from  $\tilde{H}\tilde{E}J$  of  $\mathcal{W}_{AM}^{GRR}$ . We will emphasize its importance in Section 4..

At the next to leading order  $\mathcal{W}_{FM}^{GRR} + \mathcal{W}_{AM}^{GRR}$  also contributes to sextic operator. From the discussion above we can easily infer that the leading order effective superpotential emerging from  $\mathcal{W}^{RRR}$  will be sextic in order. Similarly, integrating out the **Blue** field effectively gives  $B \sim GR/M_B + RR/M_B$  which, on replacing the **Red** field, becomes

$$B \sim \frac{GGG}{M_R M_B} + \frac{GGGG}{M_R^2 M_B} + O(G^5/M_X^4). \quad (20)$$

Therefore, we see that the leading effective operators emerging from  $\mathcal{W}_{AM}^{GRB}$  are also sextic. The effective operator emerging from  $\mathcal{W}_{AM}^{RRB}$  are septic or higher. We have listed all the sextic operators emerging from  $\mathcal{W}_{FM}^{GRR}$ ,  $\mathcal{W}_{AM}^{GRR}$ ,  $\mathcal{W}_{AM}^{RRR}$  and  $\mathcal{W}_{FM}^{GRB}$  in Table 4, and have separated out the class of sextic operators emerging only from  $\mathcal{W}_{FM}^{GRB}$  in Table 5.

### 3. Threshold enhancements and proton decay

The problem of fast proton decay in NMSGUT due to quartic terms in the effective superpotential, which has accompanied Supersymmetric GUTs from the beginning [20] can be rectified if GUT scale threshold corrections to matter fermion and MSSM Higgs vertices are incorporated [10]. Due to the large number of heavy chiral multiplets a large wave function renormalization arises driving the light Higgs fields close to dissolution ( $Z_{H,\tilde{H}} \simeq 0$ ), which modify MSSM-GUT Yukawa matching condition. Rewriting the renormalized Kinetic term

$$\mathcal{L}_{Kin} = \left[ \sum_{A,B} \tilde{f}_A^\dagger (Z_{\tilde{f}})_A^B \tilde{f}_B + f_A^\dagger (Z_f)_A^B f_B + H^\dagger Z_H H + \tilde{H}^\dagger Z_{\tilde{H}} \tilde{H} \right]_D + \dots \quad (21)$$

with canonical normalization requires the transformation [10]

$$f = U_{Z_f} \Lambda_{Z_f}^{-1/2} \tilde{f}, \quad \tilde{f} = U_{Z_f} \Lambda_{Z_f}^{-1/2} \tilde{\tilde{f}}, \\ H = \tilde{H} / \sqrt{Z_H}, \quad \tilde{H} = \tilde{\tilde{H}} / \sqrt{Z_{\tilde{H}}}, \quad (22)$$

where  $\Lambda_Z = U^\dagger Z U$  is diagonal, and the Yukawa couplings become

$$\tilde{Y}_f = \Lambda_{Z_{\tilde{f}}}^{-1/2} U_{Z_{\tilde{f}}}^T \frac{Y_f}{\sqrt{Z_{H_f}}} U_{Z_f} \Lambda_{Z_f}^{-1/2}, \quad (23)$$

and these  $\tilde{Y}_f$  are to be matched with MSSM Yukawa couplings (not the original tree level  $Y_f$ ). The crucial point is that if  $Z_H$  is small, the Spin(10) Yukawa couplings  $\{h, f, g\}_{AB}$  required to match the SM Yukawa couplings are small compared to what they would be in the absence of threshold corrections. Since the coefficients of quartic superpotential baryon decay operators depend on  $\{h, f, g\}_{AB}$  and not on  $Z_{H,\tilde{H}}$  it follows that these operators can be suppressed by one power, and the baryon decay rate by two powers of  $Z_{H,\tilde{H}}$ . For small enough  $Z_{H,\tilde{H}}$  this pushes the proton lifetime into an acceptable range, specially when one recalls that the freedom to utilize the sfermion diagonalization matrices can significantly soften [21] the  $B$  violation problem [22].

### 4. $B - L$ violating processes

Rectification of the quartic operator proton decay problem is not the only outcome of threshold corrections. Every effective operator may get corrected. Let us first look for seesaw operators which give  $B - L$  violating processes.

#### Type I seesaw:

The relevant part of the superpotential is

$$\mathcal{W} \supset \Gamma^{h\tilde{\nu}L} \tilde{H} \tilde{\nu} L + \frac{1}{2} M_{\tilde{\nu}} \tilde{\nu} \tilde{\nu}. \quad (24)$$

Integrating out the **Red** field  $\tilde{\nu}$  produces Type I seesaw quartic operator

$$O_\nu \simeq - \left( \Gamma^{h\tilde{\nu}L} \right)^2 \frac{(LH)^2}{2M_{\tilde{\nu}}}$$

which can be written in the canonical basis as

$$O_\nu \simeq - \left( \tilde{\Gamma}^{h\tilde{\nu}L} \right)^2 \frac{(\tilde{L}\tilde{H})^2}{2\tilde{M}_{\tilde{\nu}}} \quad (25)$$

where  $\Gamma_{AB}^{h\tilde{\nu}L}$  is given in eq.(14),  $\tilde{\Gamma} = \Gamma / \sqrt{Z_H \Lambda_{Z_L} \Lambda_{Z_{\tilde{\nu}}}}$  is the  $\sqrt{Z_H}$  boosted tree level Yukawa coupling for the right handed neutrinos and  $\tilde{M}_{\tilde{\nu}} = M_{\tilde{\nu}} / \Lambda_{Z_{\tilde{\nu}}}$ . Since typically [10] the wave function renormalization in  $\tilde{\Gamma}^{h\tilde{\nu}L}$  only brings the suppressed SO(10) Yukawa couplings  $h, f, g$  to ordinary MSSM magnitudes and the required size of  $\tilde{M}_{\tilde{\nu}}$  and therefore  $f_{AB}$  is fixed by the observed neutrino masses the Type I seesaw acquires an enhancement only in the sense that the  $\overline{126}$  coupling  $f_{AB}$  that enters  $\tilde{M}_{\tilde{\nu}}$  is the unboosted one : making it somewhat easier to achieve righthanded neutrino masses much smaller than the GUT scale, while the coupling  $f_{AB}$  that enters the Yukawa couplings that determine the Dirac masses

of SM fermions is the boosted one. Thus the rescalings with small  $\sqrt{Z_H}$  make the realistic Type I Seesaw more realistic and explain also the wide divergence of the right handed neutrino mass and the GUT scale.

### Type II seesaw:

On the other hand we have [9, 6]

$$\begin{aligned} W &\supset \Gamma_{AB}^{\bar{O}LL} \bar{O}_A L_B + \Upsilon^{\bar{O}} \bar{H} \bar{H} \bar{O} + \Upsilon^O H H O + M_O \bar{O} O \\ O_\nu &\simeq -\Gamma_{AB}^{\bar{O}LL} \Upsilon^O \frac{(L_A L_B)(HH)}{M_O} \\ &= -\tilde{\Gamma}_{AB}^{\bar{O}LL} \Upsilon^O \frac{(\tilde{L}_A \tilde{L}_B)(\tilde{H} \tilde{H})}{\sqrt{Z_H} M_O} \end{aligned} \quad (26)$$

where  $\Gamma_{AB}^{\bar{O}LL} = 4\sqrt{2}f_{AB}$ ,  $\Upsilon^O$  is given in eq. (9) and  $\tilde{\Gamma}^{\bar{O}LL} = \Gamma^{\bar{O}LL}/(\Lambda_{Z_L} \sqrt{Z_H})$ . This operator gets  $Z_H^{-1/2}$  enhancement after counting  $Z_H^{-1/2}$  towards bringing  $f_{AB}$  to MSSM Yukawa levels. Therefore, for small  $Z_H$ , the Type II seesaw has a better chance of making a significant contribution to the neutrino masses. This might have revived a possibility that had been dismissed with some effort [23–25], but in practice numerical fits tend to show that the boost still leaves the Type II seesaw contribution short of, or barely at, the milli-eV range.

**Nucleon decays ( $B - L \neq 0$ ):** In Table 2 the quartic effective operators are listed. The quartic baryon number violating operators for nucleon decay preserve  $B - L$ . The  $B - L$  violating processes like  $p \rightarrow \nu K^+$ ,  $n \rightarrow e^- K^+$ ,  $e^- \pi^+$  can only arise via a different mechanism. However at the quintic operator level (see the operators listed in Table 3), we have two operators which violate  $B, L$  and  $B - L$ . Since the VEV of  $H$  field picks the neutrino in the operator  $\epsilon \bar{d}_A \bar{d}_B \bar{u}_C (L_D H)$ , it allows neutron decay process with neutral leptons  $n \rightarrow \nu K^0$  only. But, in the new operator  $\epsilon \bar{d}_A \bar{d}_B \bar{d}_C (L_D \bar{H})$  the VEV of  $\bar{H}$  field picks the charged lepton allowing  $n \rightarrow e^- K^+$ . Therefore, this operator seems novel and needs to be included in the discussion of  $B - L$  violating Baryogenesis supported by SO(10) GUTs [1]. The life time for processes due to  $W_{(-1,1)}^5$  is proportional to  $|F^{I,II}|^{-2}$  (see eq.(19)). Thus we estimate that the additional presence of a light Higgs field in the operator implies that the suppression factor on the rate *relative to the suppressed threshold corrected quartic operator rate* is of order  $\langle H \rangle^2 / (M_X^2 Z_H)$  so that at best

$$\tau_{n \rightarrow \nu K^0, e^- K^+} \sim Z_H 10^{58} \text{ yrs}, \quad (27)$$

The effect of threshold corrections relative to the additional high scale suppression is thus modest. Thus in the NMSGUT the lifetime for these  $B - L$  violating modes is too large for direct detection. Its significance

for  $B - L$  mediated Baryogenesis needs detailed evaluation in view of the additional possibilities for CP violation, or exceptional parameter combinations in these operators.

## 5. Conclusions

We have presented the gist of our results concerning the effective superpotential emerging from NMSGUT up to terms sextic in the light fields. We have also discussed how the GUT scale threshold corrections which rescue [10] the phenomenologically successful NMSGUT [4, 3, 5, 9, 6] impact various processes including  $B - L$  violating processes. We noted that the Type II seesaw neutrino masses are boosted sufficiently by threshold corrections to require inspection for significance in detailed fits. Examples of expressions for the coefficients of the effective operators were given here but detailed results will be presented elsewhere [18] due to their length. As is always true in Supersymmetric R-parity preserving GUTs, since the survival hypothesis fails due to intermediate scales resulting in large pseudo-goldstone multiplets ruining gauge coupling unification [3], the  $B - L$ ,  $SU(2)_R$  violation scales are required to be quite near the GUT scale. Hence the novel higher dimension operators, failing the discovery of anomalously enhanced coefficients for some special case, are severely suppressed, in spite of the enhancement by the near dissolution values ( $Z_{H,\bar{H}} \ll 1$ ) of the light Higgs renormalization factors. A detailed evaluation of the feasibility of  $B - L$  violating Baryogenesis will be considered once a fully satisfactory fit taking account of off diagonal coupling matching at the Susy breaking scale is completed [18].

## Appendix

In this Appendix we present tables of the different types of effective superpotential operators, upto sextic order, that we found by integrating out heavy fields using the superpotential (momentum independent) equations of motion.

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$\mathcal{W} = \mathcal{O}^{d=4}/M_X$					
	Operators $\mathcal{O}_i$	$B$	$L$	$B-L$	Enh.
1	$\epsilon Q_A Q_B Q_C L_D$	1	1	0	No
2	$Q_A L_B \bar{u}_C \bar{e}_D$	0	0	0	No
3	$Q_A Q_B \bar{u}_C \bar{d}_D$	0	0	0	No
4	$\epsilon \bar{e}_A \bar{u}_B \bar{u}_C \bar{d}_D$	-1	-1	0	No
5	$(L_A L_B)(HH),$ $(L_A H)(L_B H)$	0	2	-2	$1/Z_H$
6	$(\bar{H}H)(\bar{H}H),$ $(HH)(\bar{H}\bar{H})$	0	0	0	$1/Z_H^2$

**Table 2.** Quartic Effective Superpotential Operators from NMSGUT.

$\mathcal{W} = \mathcal{O}_i^{d=5}/M_X^2$					
	Operators $\mathcal{O}_i$	$B$	$L$	$B-L$	Enh.
1	$\epsilon \bar{d}_A \bar{d}_B \bar{u}_C (L_D H)$	-1	1	-2	$1/\sqrt{Z_H}$
2	$\epsilon \bar{d}_A \bar{d}_B \bar{d}_C (L_D \bar{H})$	-1	1	-2	$1/\sqrt{Z_H}$
3	$(Q_A L_B)(L_C H) \bar{d}_D,$ $(L_A L_B)(Q_C H) \bar{d}_D$	0	2	-2	$1/\sqrt{Z_H}$
4	$(L_A L_B)(L_C H) \bar{e}_D$	0	2	-2	$1/\sqrt{Z_H}$
5	$(L_A H)(\bar{H}\bar{H}) \bar{e}_B,$ $(L_A \bar{H})(H\bar{H}) \bar{e}_B$	0	0	0	$1/\sqrt{Z_H^3}$
6	$(Q_A \bar{H})(HH) \bar{u}_B,$ $(Q_A H)(H\bar{H}) \bar{u}_B$	0	0	0	$1/\sqrt{Z_H^3}$
7	$(Q_A \bar{H})(H\bar{H}) \bar{d}_B,$ $(Q_A H)(\bar{H}\bar{H}) \bar{d}_B$	0	0	0	$1/\sqrt{Z_H^3}$

**Table 3.** Quintic Effective Superpotential Operators from NMSGUT.

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$\mathcal{W} = \mathcal{O}_i^{d=6}/M_X^3$				
	Operators $\mathcal{O}_i$	$B$	$L$	$B-L$
1	$(\epsilon \bar{d}_{A_1} \bar{d}_{B_1} \bar{u}_{C_1})(\epsilon \bar{d}_{A_2} \bar{d}_{B_2} \bar{u}_{C_2})$	-2	0	-2
2	$(\epsilon \bar{d}_{A_1} \bar{d}_{B_1} \bar{u}_{C_1})(Q_{A_2} L_{B_2}) \bar{d}_{C_2}$	-1	1	-2
3	$(\epsilon \bar{d}_{A_1} \bar{d}_{B_1} \bar{u}_{C_1})(L_{A_2} L_{B_2}) \bar{e}_{C_2}$	-1	1	-2
4	$(\epsilon \bar{d}_A \bar{d}_B \bar{u}_C)(\bar{H} \bar{H}) \bar{e}_D$	-1	-1	0
5	$(Q_A L_B)(Q_C L_D) \bar{d}_E \bar{d}_F$	0	2	-2
6	$(Q_A L_B)(L_C L_D) \bar{d}_E \bar{e}_F$	0	2	-2
7	$(Q_A L_B)(\bar{H} \bar{H}) \bar{d}_C \bar{e}_D$	0	0	0
8	$(L_A L_B)(L_C L_D) \bar{e}_E \bar{e}_F$	0	2	-2
9	$(L_A L_B)(\bar{H} \bar{H}) \bar{e}_C \bar{e}_D$	0	0	0
10	$(\bar{H} \bar{H})(\bar{H} \bar{H}) \bar{e}_A \bar{e}_B$	0	-2	2
11	$(L_A H)(L_B H)(\bar{H} H)$	0	2	-2
12	$(Q_A \bar{H})(L_B H) \bar{u}_C \bar{e}_D$	0	0	0
13	$\epsilon(Q_A Q_B)(Q_C \bar{H})(L_D H)$	1	1	0
14	$(L_A H)(L_B \bar{H})(H H),$ $(L_A H)(L_B H)(H \bar{H})$	0	2	-2

**Table 4.** Sextic Effective Superpotential Operators from NMSGUT.

$\mathcal{W} = \mathcal{O}_i^{d=6}/M_X^3$				
	Operators $\mathcal{O}_i$	$B$	$L$	$B-L$
1	$(H \bar{H}) Q_A L_B \bar{u}_C \bar{e}_D$	0	0	0
2	$(H \bar{H}) Q_A Q_B \bar{u}_C \bar{d}_D$	0	0	0
3	$(H H) Q_A Q_B \bar{u}_C \bar{u}_D$	0	0	0
4	$(\bar{H} \bar{H}) Q_A Q_B \bar{d}_C \bar{d}_D$	0	0	0
5	$(H \bar{H}) \epsilon \bar{e}_A \bar{u}_B \bar{u}_C \bar{d}_D$	-1	-1	0
6	$(H \bar{H})(\bar{H} H)(\bar{H} H)$ $(H \bar{H})(H H)(\bar{H} \bar{H})$	0	0	0

**Table 5.** Sextic Effective Superpotential Operators from  $\mathcal{W}^{GRB}$  part of NMSGUT.

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